Problems for Review

13.2 7, 10, 12

3.3 12, 14

Fall 2017 Test 1 1, 2, 3, 4

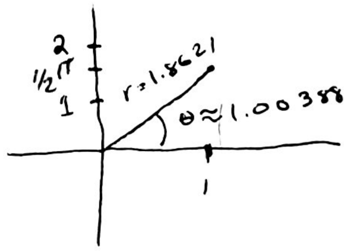
13.2

#7) $z = 1 + \frac{1}{2}\pi j$

$z = \sqrt{1^2 + (\frac{\pi}{2})^2} = 1.8621$

$\tan^{-1}(\pi/2) \approx 1.00388$

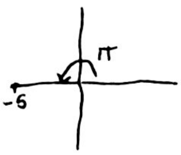
$$\boxed{z = 1.8621\left[\cos(1.00388) + j\sin(1.00388)\right]}$$



10

a) $z = -5 + j0$

$\text{Arg}(z) = \tan^{-1}\left(\frac{0}{5}\right) = \tan^{-1}(0) = 0$

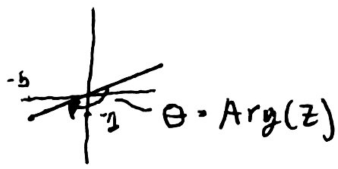


$-\pi < \text{Arg}(z) \leq \pi$

$$\boxed{\text{Arg}(z) = \pi}$$

b) $z = -5 - j$

$\tan^{-1}\left(\frac{-1}{-5}\right) = \tan^{-1}\left(\frac{1}{5}\right) \approx .197396$



$\text{Arg}(z) = .197396 - \pi$

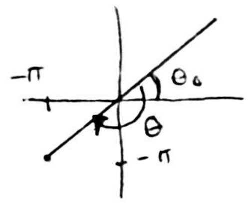
$$\boxed{\text{Arg}(z) = -2.9442}$$

13.2

12) $z = -\pi - j\pi$ what is Arg(z)

$$\text{Arg}(z) = \tan^{-1}\left(\frac{-\pi}{-\pi}\right) = \tan^{-1}(1) = \frac{\pi}{4} = \theta_0$$

plot z to check region



$$\text{Arg}(z) = \theta_0 - \pi$$
$$= \frac{\pi}{4} - \pi$$
$$\boxed{\text{Arg}(z) = -\frac{3\pi}{4}}$$

13.3

12) $f(z) = \dfrac{z-2}{z+2}$ $z_0 = 8j$ find $\text{Re}\, f(z)$, $\text{Im}\, f(z)$

Step 1 $f(z_0) = \dfrac{8j-2}{8j+2} = \dfrac{8j-2}{8j+2}\,\dfrac{(8j-2)}{(8j-2)} = \dfrac{-16j-64+4-16j}{\cancel{16j}-64-4-\cancel{16j}}$

$$f(z_0) = \frac{-60-32j}{-68}$$

$$\text{Re}\, f(z) = \frac{-60}{-68} = \boxed{\frac{60}{68}}$$

$$\text{Im}\, f(z) = \frac{-32}{-68} = \boxed{\frac{32}{68}}$$

13.3

14) Determine if $f(z)$ is continouse if $f(z_0)=0$ and $z_0=0$ are given.

$z = x + jy$

$0 = x + jy$

$x_0 = 0$

$y_0 = 0$

$f(z) = (\operatorname{Re} z^2)/|z|$

$$f(z) = \frac{x^2 - y^2}{\sqrt{x^2+y^2}}$$

$z^2 = x^2 - y^2 + 2jxy$

$|z| = \sqrt{x^2+y^2}$

Path 1

$$\lim_{y=y_0,\, x\to x_0} \frac{x^2-y^2}{\sqrt{x^2+y^2}} = \frac{x^2-0^2}{\sqrt{x^2+0^2}} = \frac{x^2}{x} = x = 0 \checkmark$$

Path 1 works as

$$\lim_{y=y_0;\, x\to x_0} f(z) = f(z_0)$$

Path 2

$$\lim_{x=x_0,\, y\to y_0} \frac{x^2-y^2}{\sqrt{x^2+y^2}} = \frac{0-y^2}{\sqrt{0^2+y^2}} = \frac{-y^2}{y} = -y = 0$$

Path 2 work as

$$\lim_{x=x_0,\, y\to y_0} f(z) = f(z_0)$$

$f(z)$ can be assumed to be ~~com~~ continouse.

Test1 Q 1 Fall 2017

1. Given the complex polynomial $z^3 - 2 - j2 + e^{j\pi} = 0$

(a) Detemine the roots (results must be in rectangular)

(b) Plot the roots on complex Plane

$$z^3 - 2 - j2 + e^{j\pi} = 0$$

$$e^{j\pi} = -1 = 1(\cos\pi + j\sin(\pi))$$

$$= 1(-1 + j0)$$

$$e^{j\pi} = -1 = -1$$

$$z^3 - 2 - j2 - 1 = 0$$

$$z^3 = 3 + 2j$$

$$|r| = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$$

$$\tan^{-1}(2/3) = .588003 + 2\pi k, \quad k \in \mathbb{I}$$

$$z^3 = \sqrt{13}\, e^{j(.588003 + 2\pi k)}$$

$$z = \sqrt[3]{3.605}\, e^{j\left(\frac{.588003}{3} + \frac{2}{3}\pi k\right)}$$

$$\cancel{z_{k=0} = 1.53341\, e}$$

$$z_{k=0} = 1.53341\left[\cos\left(\frac{.588003}{3}\right) + j\sin\left(\frac{.588003}{3}\right)\right]$$

$$= \cancel{1.50405 + .298622}$$

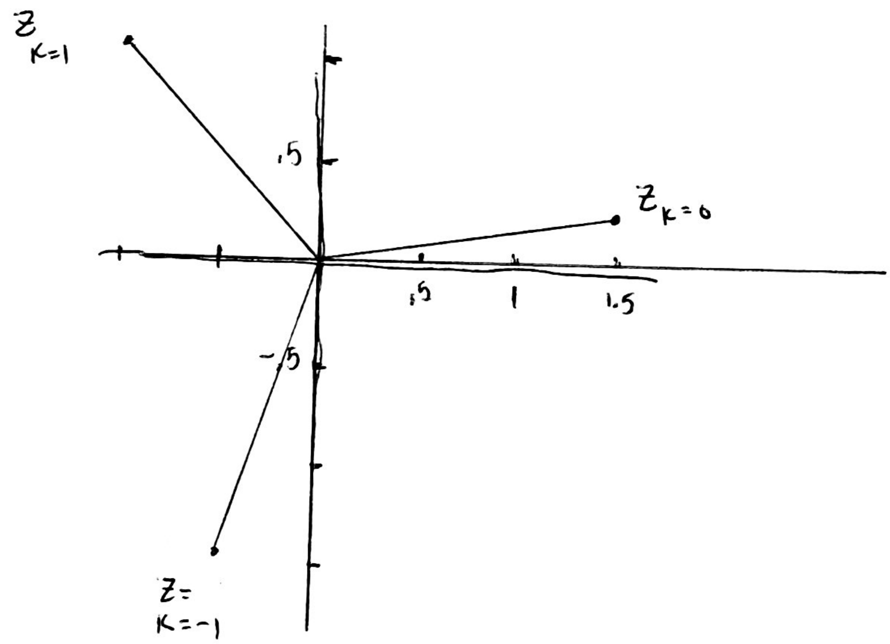
$$= 1.50405 + .298628j$$

$$z_{k=1} = 1.53341\left[\cos\left(\frac{.588003}{3} + \frac{2}{3}\pi\right) + j\sin\left(\frac{.588003}{3} + \frac{2}{3}\pi\right)\right]$$

$$= -1.01064 + 1.15323\, j$$

$$z_{k=-1} = 1.53341\left[\cos\left(\frac{.588003}{3} - \frac{2}{3}\pi\right) + j\sin\left(\frac{.588003}{3} - \frac{2}{3}\pi\right)\right]$$

$$= -.493404 - 1.45186 \cdot j$$



#2 Find all solutions for

$$\sin z - 2 + j4 - e^{j\frac{\pi}{4}} = 0$$

$$e^{j\pi/4} = 1[\cos \tfrac{\pi}{4} + j \sin \tfrac{\pi}{4}]$$

$$= \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$$

$$\sin z - 2 + j4 - \frac{\sqrt{2}}{2} \mp j\frac{\sqrt{2}}{2} = 0$$

$$\sin z = 2 + \frac{\sqrt{2}}{2} - j4 + j\frac{\sqrt{2}}{2}$$

$$\sin z = \frac{\sqrt{2}}{2} + 2 - j\left(4 - \frac{\sqrt{2}}{2}\right)$$

$$r = \sqrt{\left(\frac{\sqrt{2}}{2} + 2\right)^2 + \left(4 - \frac{\sqrt{2}}{2}\right)^2} \approx 4.26281$$

$$\tan^{-1}\left(\frac{4 - \frac{\sqrt{2}}{2}}{2 + \frac{\sqrt{2}}{2}}\right) \approx .882721 + 2\pi k$$

$$\sin z = 4.26281 e^{j(.882721 + 2\pi k)}$$

$$\frac{1}{2j}\left(e^{jz} - e^{-jz}\right) = 4.26281 e^{j(.882721 + 2\pi k)}$$

$$\ln\left\{ e^{jz} - e^{-jz} = 2j\, 4.26281\, e^{j(.882721 + 2\pi k)} \right\}$$

$$jz + jz = \ln(8.525 j) + j(.882721 + 2\pi k) \qquad k \in \mathbb{Z}$$

$$\frac{2jz}{2j} = \frac{\ln(8.525 j) + j(.882721 + 2\pi k)}{2j}$$

$$\boxed{z = \frac{\ln(8.525 j)}{2j} + \frac{(.882721 + 2\pi k)}{2}}$$

#3 Given an entire function $f(z) = U(x,y) + jV(x,y)$ where $z = x + jy$ and

$U(x,x) = \cos(6x)\cosh(3y)$

Determine 6 and the harmonic conjugate $V(x,y)$

$\cancel{U = \cos 6x \cos(3y)}$

$U = \cos 6x \cosh(3y)$

$U_x = 6 - \sin 6x \cosh(3y)$

$U_y = \cos 6x \sinh(3y) 3$

$\cancel{V_y = U_x = -6 \sin 6x \cosh 3}$

$V_y = U_x = -6 \sin 6x \cosh(3y)$

$V_x = -U_y = -3 \cos 6x \sinh(3y)$

$$U(x,y) = \int V_y \, dy = \int -6 \sin 6x \cosh(3y) dy$$

$$= -6 \sin 6x \int \cosh(3y) dy$$

$$V(x,y) \quad = -6 \sin 6x \frac{1}{3} \sinh 3y + h(x)$$

$$V_x(x,y) = -\frac{6}{3} \sinh(3y) \cos(6(x)) + \frac{d h(x)}{dx}$$

$$-3 \cancel{\cos} 6x \cancel{\sinh(3y)} = -\frac{6}{3} \cancel{\sinh(3y)} \cancel{\cos} 6x + \cancel{\frac{d h(x)}{dx}}^{0}$$

$$-3 = \frac{-6}{3}$$

$$-9 = -6$$

$$6 = 9$$

$$h(x) = \int h'(x) dx$$

$$= \int 0 \, dx$$

$$= 0x + C$$

$$h(x) = C$$

$$U = \cos(9x)\cosh(3y)$$

$$v = -9\sin 9x \cdot \frac{1}{3}\sinh(3y)$$

$$\boxed{\begin{aligned} \vec{v} &= -3\sin(9x)\sinh(3y)+c \\ b &= 9 \end{aligned}}$$

#4

$e^{z} + 4 = 2e^{j\pi}$ find all solutions and graph five of them in the complex plane

$$e^{z} + 4 = 2e^{j\pi}$$

$$2e^{j\pi} = 2[\cos(\pi)^{\,-1} + j\,\cancel{\sin(\pi)}^{\,0}]$$
$$= -2$$

$$e^{z} + 4 = -2$$
$$\quad -4 \quad\; -4$$

$$e^{z} = -6$$

$$\ln e^{z} = \ln(-6)$$

$$-6 + 0j$$

$$r = \sqrt{6^2 + 0^2} = 6$$

$$z = \ln(-6)$$

$$\tan^{-1}\left(\frac{0}{-6}\right) = \pi + 2\pi K, \quad K \in \mathbb{Z}$$

$$z = \ln|z| + j\,\theta_0$$

$$z = \ln(6) + j(\pi + 2\pi k), \quad K \in \mathbb{Z}$$